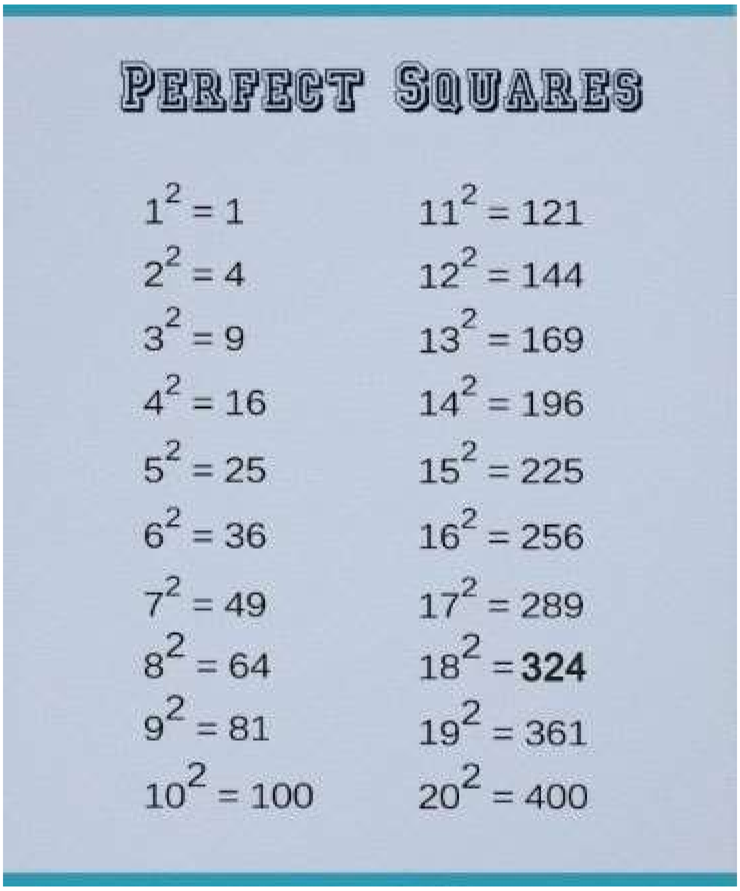
Novena sesión del curso 2021-2022: **Viernes 22 de abril de 2022, de 18:15 a 19:45 horas.**

- 4º de ESO (Aula 4 del Edificio de Matemáticas): **Aproximando raíces cuadradas sin calculadora**, por Pedro J. Miana (Departamento de Matemáticas, Universidad de Zaragoza).

1) Las calculadoras humanas

https://www.youtube.com/watch?v=YNM2qkOts1o

2) Los 20 primeros cuadrados perfecto, y trucos de los cuadrados



3) Sobre cómo calcular raíces cuadradas:

https://www.youtube.com/watch?v=TNxDukK9Vrk

4) ENIAC 1946

https://www.youtube.com/watch?v=32o4qcYbWMA

http://www.conec.es/historia/%E2%80%A8el-primer-ordinador-del-mon-compleix-anys/

https://www.youtube.com/watch?v=PVcA9NGEwpI (Figuras ocultas, 2016)

Taller de talento Matemático

4 de la ESO

22/04/2022 (Pedro J. Miana)

18:15 / 19:45

Cuadrados Perfectos

$1^2 = 1$
$2^2 = 4$
$3^2 = 9$
$4^2 = 16$
$5^2 = 25$
$6^2 = 36$
$7^2 = 49$
$8^2 = 64$
$9^2 = 81$
$10^2 = 100$

$11^2 = 121$
$12^2 = 144$
$13^2 = 169$
$14^2 = 196$
$15^2 = 225$
$16^2 = 256$
$17^2 = 289$
$18^2 = 324$
$19^2 = 361$
$20^2 = 400$

$21^2 = 441$
$22^2 = 484$
$23^2 = 529$
$24^2 = 576$
$25^2 = 625$
$26^2 = 676$
$27^2 = 729$
$28^2 = 784$
$29^2 = 841$
$30^2 = 900$

$31^2 = 961$
$32^2 = 1024$
$33^2 = 1089$
$34^2 = 1156$
$35^2 = 1225$
$36^2 = 1296 = 6^4$
$37^2 = 1369 =$
$38^2 = 1444$
$39^2 = 1521$
$40^2 = 1600$

Cuadrados y cubos.

324 $\qquad (16)^2 = 256$

$$(18)^2 = (16+2)^2 = 16^2 + 2\cdot 2\cdot 16 + 2^2 =$$
$$= 256 + 64 + 4 = 256 + 68$$
$$= 324$$

- $(19)^2 = (20-1)^2 = 400 - 40 + 1 = 361$

- $(29)^2 = (30-1)^2 = 900 - 60 + 1 = 841$

- $(38)^2 = (40-2)^2 = 1600 - 160 + 4 = 1444$

Cubos

$$(19)^3 = (20-1)^3 = 20^3 - 3\cdot 20^2 + 3\cdot 20 - 1^3 =$$
$$= 8000 - 1200 + 60 - 1 =$$
$$= 6800 + 60 - 1 = 6859$$

$$(a-b)^3 = a^3 - 3a^2b + 3a\cdot b^2 - b^3$$
$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(26)^3 = (25+1)^3 = (25)^3 + 3\cdot 25^2 + 3\,25 + 1 =$$
$$= 15625 + 3\cdot 625 + 75 + 1$$
$$= 15625 + 1875 + 76 = 17576$$

$$(31)^3 = (30+1)^3 = 30^3 + 3\cdot 30^2 + 3\cdot 30 + 1 =$$
$$= 27000 + 3\cdot 900 + 90 + 1 = 27000 + 2700 + 91$$
$$= 29791$$

$\sqrt{68{,}04{,}}$ | $82{,}48$

$8 \times 8 = 64$

$162 \times 2 = 324$

$1644 \times 4 = 6576$

$824 \times 2 = 1648$

$16488 \times 8 = 131904$

64

404

324

08000

6576

142400

131904

10496

$\sqrt{6804} = 82'486$

Raiz cuadrada de 16641

$\sqrt{1,66,41}$	129
-1	$22 \times 2 = 44$
$0\,66$	$249 \times 9 = 2241$
-44	
2241	
2241	

$\sqrt{166}$ $\quad \sqrt{144} = 12 \quad 12\_$

$\sqrt{169} = 13$

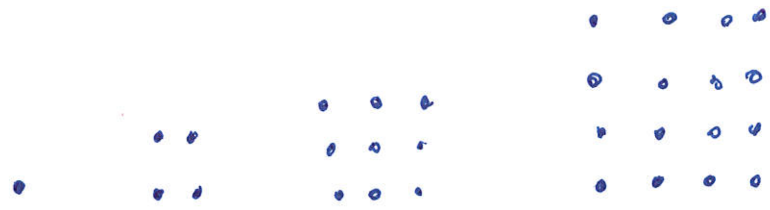
$\sqrt{10.000} = \sqrt{10^4} = 100$ | $12\cdot 10 = (120)^2 = 14400$

$\sqrt{400.00} = \sqrt{4\cdot 10\cdot 000} = 2\cdot 100$ | $(13\cdot 10) = (130)^2 = 16900$

$$(129)^2 = (130-1)^2 = (130)^2 - 2\cdot 1\cdot 30 + 1 =$$

$$= 16900 - 260 + 1 = 16640 + 1 = 16641$$

Otra aproximación por cuadrados. ENIAC 1946



$\underline{1}$ $4 = 1 + \underline{3}$ $4 + \underline{5} = 9$ $16 = 9 + \underline{7}$ $25 = 16 + \underline{9}$ $36 = 25 + 11$

$$n^2 = 1 + 3 \ldots\ldots + 2n - 1$$

Inducción $n = 1$, $1 = 1$

$$n^2 = 1 + 3 + 2n - 1$$

$$(n+1)^2 = n^2 + 2n + 1 = 1 + 3 + \cdot + 2n - 1 + 2n + 1$$

$\sqrt{28}$

$$\sqrt{25} < \sqrt{28} < \sqrt{36}$$

$$5 < \sqrt{28} < 6$$

$$5 < 5{,}27 < 6.$$

$$\sqrt{28} \simeq 5 + \frac{3}{11} = 5 + 0{,}2727 = 5{,}27$$

$$\sqrt{a+b} \not\leq \sqrt{a} + \sqrt{b}$$

$$\sqrt{28} = \sqrt{5^2 + 3}$$

$$28 < 25 + 11$$

$$25 + 3 < 25 + 11$$

$$\frac{3}{11} < 1 \text{ obvio.}$$

Teorema Sea $x \in [0, 2a+1]$ entonces.

$$a \leq a + \frac{x}{2a+1} \leq \sqrt{a^2+x} \quad , \quad x \in [0, 2a+1]$$

-demostración-

$$a + \frac{x}{2a+1} \leq \sqrt{a^2+x}$$

$$\cancel{a^2} + \frac{2ax}{2a+1} + \frac{x^2}{(2a+1)^2} \leq \cancel{a^2} + x$$

$$\frac{2a}{2a+1} + \frac{x}{(2a+1)^2} \leq 1$$

$$\cancel{2a} + \frac{x}{2a+1} \leq \cancel{2a} + 1$$

$$\frac{x}{2a+1} \leq 1,$$

$$x \leq 2a+1.$$

$$(n-1)^2 \leq x < n^2$$

$$n-1 \leq \sqrt{x} < \sqrt{n^2} = n$$

$$n-1 \leq \sqrt{(n-1)^2 + \alpha} < n$$

$$n-1 + \frac{\alpha}{2n-1} \leq \sqrt{x}$$

$$(n-1) + \frac{x-(n-1)}{2n-1} \leq \sqrt{x}$$

$$f(x) = \sqrt{a^2+x} - \left(a + \frac{x}{2a+1}\right)$$

$$f^2(x) = a^2 + x - 2\sqrt{a^2+x}\left(a + \frac{x}{2a+1}\right) + \left(a + \frac{x}{2a+1}\right)^2$$

$$\leq (a^2+x) - 2\left(a + \frac{x}{2a+1}\right)^2 + \left(a + \frac{x}{2a+1}\right)^2$$

$$\leq a^2 + x - \left(a + \frac{x}{2a+1}\right)^2 = a^2 + x - \left(a^2 + \frac{2xa}{2a+1} + \frac{x^2}{(2a+1)^2}\right)$$

$$= x - \frac{2xa}{2a+1} - \frac{x^2}{(2a+1)^2} = \frac{x}{2a+1}\left(\cancel{2a} + 1 - \cancel{2a} - \frac{x}{2a+1}\right)$$

$$= \frac{x}{2a+1}\left(1 - \frac{x}{2a+1}\right)$$

Ejemplo.

- $75 = 64 + \frac{11}{17}$

$\sqrt{75} = 8,66$

$\sqrt{75} \cong 8 + \frac{11}{17} = 8,064$

$64 \leq 75 \leq 81$

$81 - 64 = 17$

- 645

$25^2 = 625 \leq 645 \leq 26^2 = 676$

$25 \leq \sqrt{645} \leq 26$

$676 - 625 = 51$

$645 - 625 = 020$

$\sqrt{645} \approx 25 + \frac{20}{51} = 25 + 0'3921 = 25,3921$

$\sqrt{645} = 25,3968$

- 1487

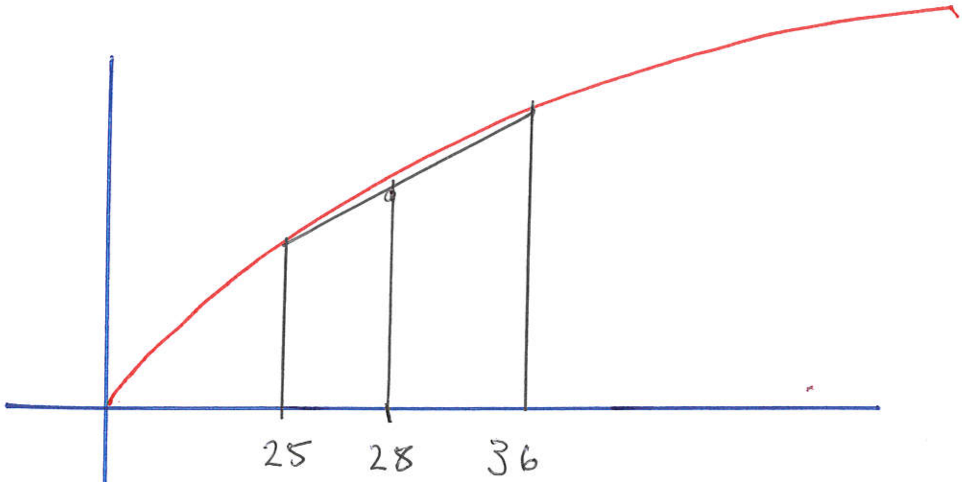
$38^2 = 1444 \leq 1487 \leq 1521 = 39^2$

$1521 - 1444 = 77$

$1487 - 1444 = 043$

$\sqrt{1487} = 38,566$

$\sqrt{1487} \cong 38 + \frac{43}{77} = 38 + 0'558 = 38,568$



interpolacion lineal $p_1 = 5 + \frac{x-25}{11}$

$\sqrt{28}$ $p_1(x)$

Digito a Digito

Vamos a utilizar otro procedimiento, como flotante.
en el intervalo $(0,100) = (0,10^2)$

$\sqrt{43}$, $36 \lneq 43 < 49$

$6 < \sqrt{43} < 7$

$\sqrt{43} = 6 + \frac{d_1}{10}$, $43 = (6+\frac{d_1}{10})^2 = 36 + 2\cdot 6\cdot\frac{d_1}{10} + \cancel{\frac{d_1^2}{100}}$

$\geq 36 + \frac{6}{5} d_1$, $43-36 = 7$, $7 \geq \frac{6}{5} d_1$

$\frac{35}{6} \geq d_1$

$\left[\frac{35}{6}\right] = \underline{\underline{5}}$

$\sqrt{43} = 6,5 + \frac{d_2}{100}$

$43 = (6,5 + \frac{d_2}{100})^2 = 42,25 + 2\cdot 6'5 \cdot \frac{d_2}{100} + \cancel{(\frac{d_2}{100})^2}$

$0'75 = 13 \cdot \frac{d_2}{100}$, $\left[\frac{75}{13}\right] = d_2$, $d_2 = 5$

$\sqrt{43} = 6,55 + \frac{d_3}{1000}$

$43 = (6,55 + \frac{d_3}{10^3})^2 = (6,55)^2 + 2\cdot 6,55 \frac{d_3}{10^3} + \cancel{(\frac{d_3}{10^3})^2}$

$43 \cong 42,9025 + \frac{13,1}{10^3} d_3$

$0,0975 \cong \frac{13,1}{10^3} d_3$ $d_3 = \left[\frac{97'5}{13'1}\right] = 7$

$\sqrt{43} \approx 6,557$

$\sqrt{43} \simeq 6,557438$